

Jam Guard

GPS Anti-Jamming Solution



Senior Design Project

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1 May 2026

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<https://jamguard.net>

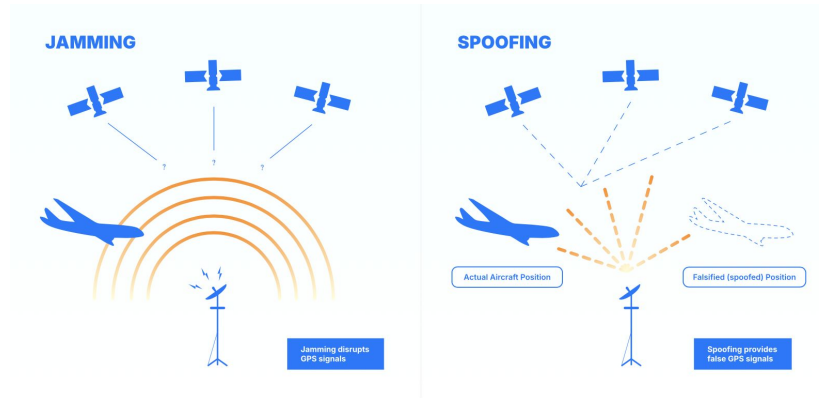
- Background
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- Existing Solutions
- Value Proposition
- Marketing & Engineering Requirements
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- Global Navigation Satellite System (GNSS) was developed by the military in the 1970's with the first satellite launched in 1978
- GPS is the U.S. owned iteration with 31 Satellites, but others exist (Galileo-E.U, BeiDou-China)
- Operate on multiple frequencies L1 (1575.42 ± 7.5 MHz), L2 (1227.60 ± 5.5 MHz), and L5 (1176.45 ± 6.25 MHz)



Background (Contd.)

- GPS signals reach Earth at very low power levels (typically -130 dBm)
- Broadband Noise Jammers (most common) overwhelm GPS signal
- Other methods, spoofing and meaconing are more sophisticated forms of signal disruption



Modern Examples of Jamming

Gulf Region Jamming (Oct 2025)

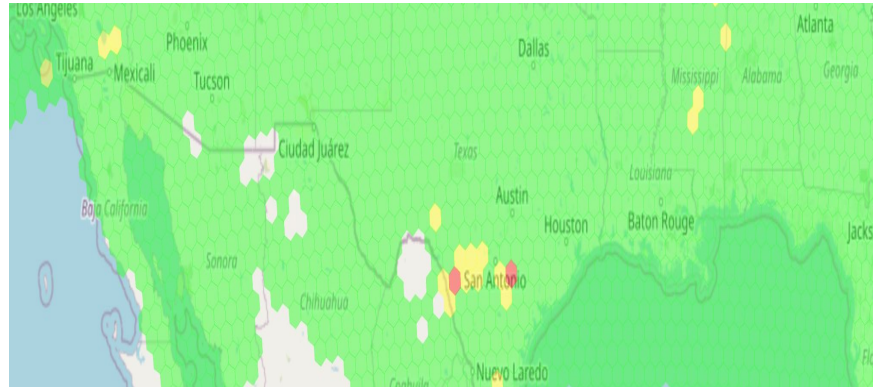
- Mass gps jamming/interference affected over 1000 vessels

Border Gps disruption (Current)

- Border Patrol faces jamming issues at the border, despite advancement in technology

Civil Aviation -Tartu Airport Estonia (2024-25)

- gps jamming resulted in unsafe landing conditions



Problem Statement

Low-cost wideband jammers can overwhelm GPS signals, preventing receivers from maintaining a reliable position fix. This project focuses on the elimination of jamming interference to ensure a continuously working GPS system. This issue impacts critical users such as transportation, emergency services, defense, and autonomous technologies, where jamming—both accidental and intentional—can disrupt navigation and safety. A robust, low-cost solution is needed to maintain GPS functionality in these environments.

Existing Solution Methods

Spatial filtering

- Spatial filtering uses multiple elements(antennas) and emphasizes phase control to separate signals based on the direction they come from
- Beamforming directs antenna gain toward GPS signals
- Null-Steering suppresses interference by creating nulls toward the direction of a jammer

Temporal filtering

- Temporal filtering process a signal based off time or frequency of the signal

Pre/Post Correlation

- pre-correlation filtering removes broadband noise and interference from a signal
- post-correlation filtering enhances signal quality by removing jamming effects and any residual noise

Existing Solutions

- Tualaj 4200 Mini GPS/GNSS Anti-Jam CRPA System
 - \$ Quote required/Military Grade
(CRPA = Controlled Reception Pattern Antenna)
- Anti-jamming device infiniDome GPSdome-SunStone – dual-band GNSS signal protection, compact design
 - \$ 2700
- Orolia GPSdome 1.02B Anti-Jammer
 - \$ Quote required- only non-ITAR
(ITAR= International Traffic of Arms Regulation)



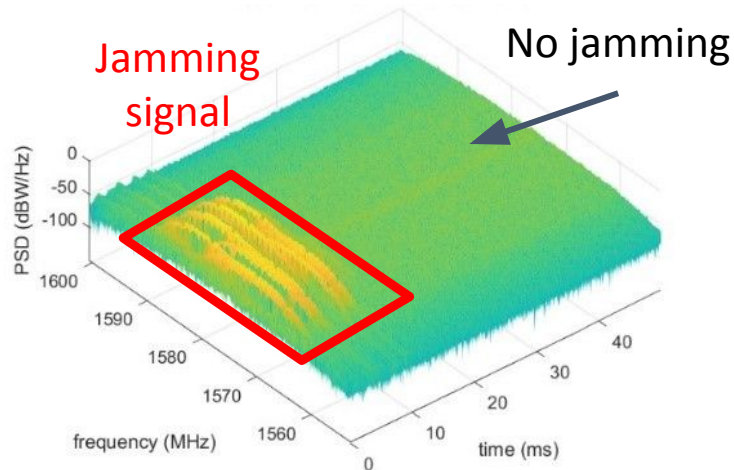
Tualaj 4200 mini



Orolia GPSdome

Value Proposition

Jam Guard helps GPS-reliant systems operate reliably in jamming environments by suppressing interfering RF signals, allowing receivers to maintain functionality, unlike conventional systems that lose accuracy or fail under interference.



Marketing Requirements

- MR 1** Device shall reliably detect GPS broadband jamming
- MR 2** Device shall integrate easily with existing GPS receivers
- MR 3** Device shall maintain usable GPS under broadband jamming
- MR 4** Device shall be compact and portable
- MR 5** Device shall operate efficiently using a battery
- MR 6** Device shall be easy to install and operate

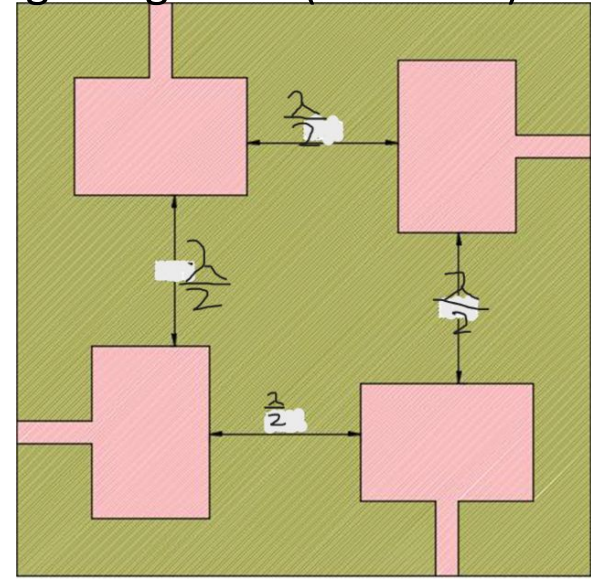
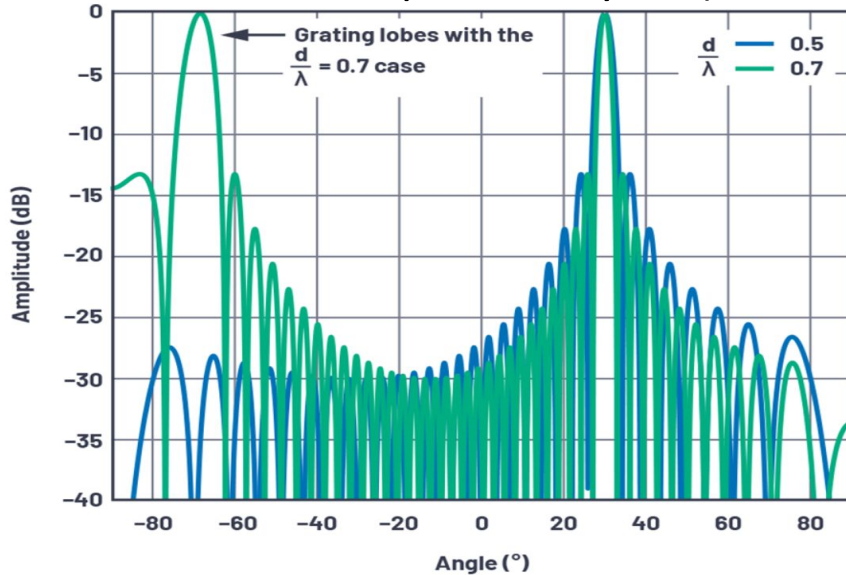
Engineering Requirements

- ER 1.1** Device must detect jamming with $\geq 95\%$ probability at Jamming Signal Ratio ≥ 10 dB
- ER 1.2** Device must trigger jamming alert in < 500 ms
- ER 2.1** Device must support GPS L1 ($1,575.42 \text{ MHz} \pm 7.5 \text{ MHz}$)
- ER 2.2** Device must have a return loss > 10 dB
 - ER 3** Device must provide ≥ 20 dB null-forming suppression at $1,575.42 \pm 15.34 \text{ MHz}$
 - ER 4** Device's PCB footprint must $\leq 15 \times 15$ cm and must weigh less than 600 g
 - ER 5** Device must consume average current ≤ 200 mA and maximum current ≤ 300 mA
- ER 6.1** Device must utilize a u.FI connector for antenna connection
- ER 6.2** Device must be operational after $\leq 100 \mu\text{s}$ from turning on

Solution Theory (Array)

Phased Antenna Array

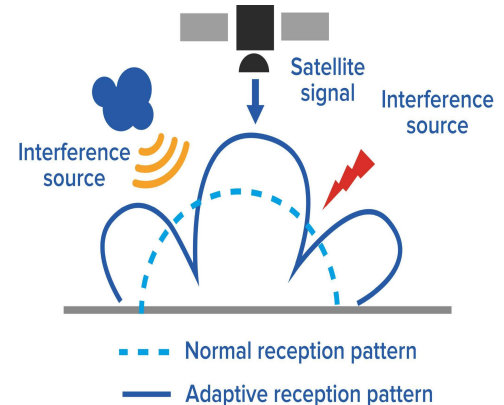
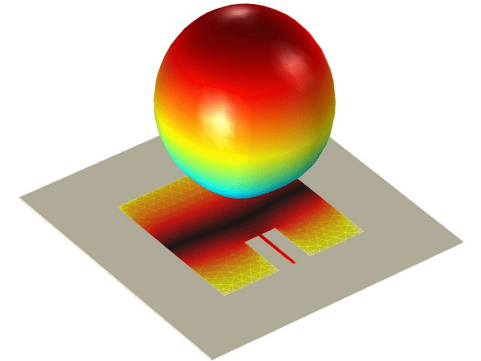
- Electronic Antenna Array movement through Phase Shifter
- Changes lobe size/direction based off Phase and Magnitude
- Elements Separated by $.5\lambda$ (Standard) to prevent grating lobes (9.525 cm)



Radiation Pattern

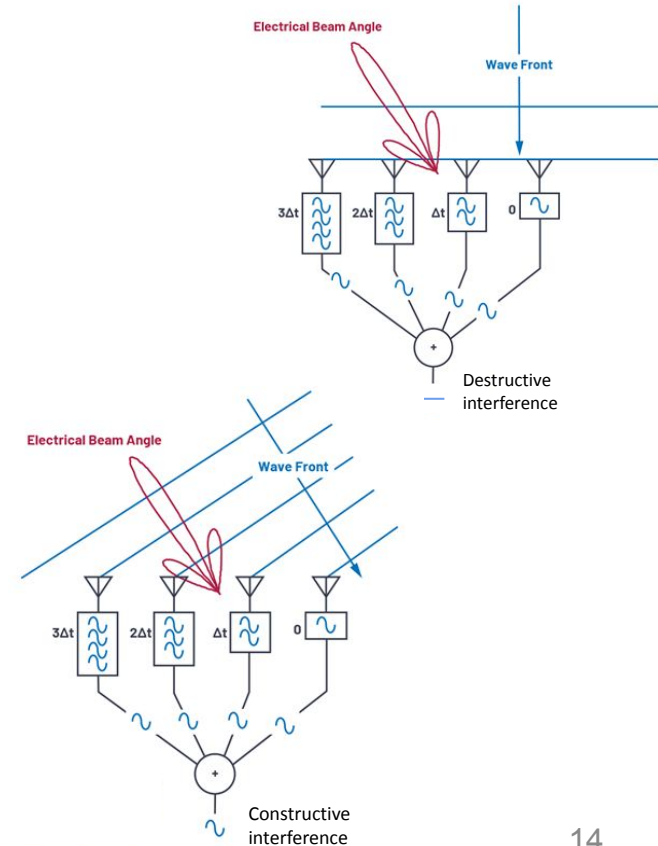
- A radiation pattern shows how an antenna sends energy into space
It tells us **where the signal is strong or weak**
- **Main Lobe**
Direction which has the **highest signal power**
- **Sidelobes**
Smaller, unwanted radiation in other directions
- **Nulls**
Directions where **signal is minimized or cancelled**

- **Why it matters**
Determines **coverage, interference, and signal quality**

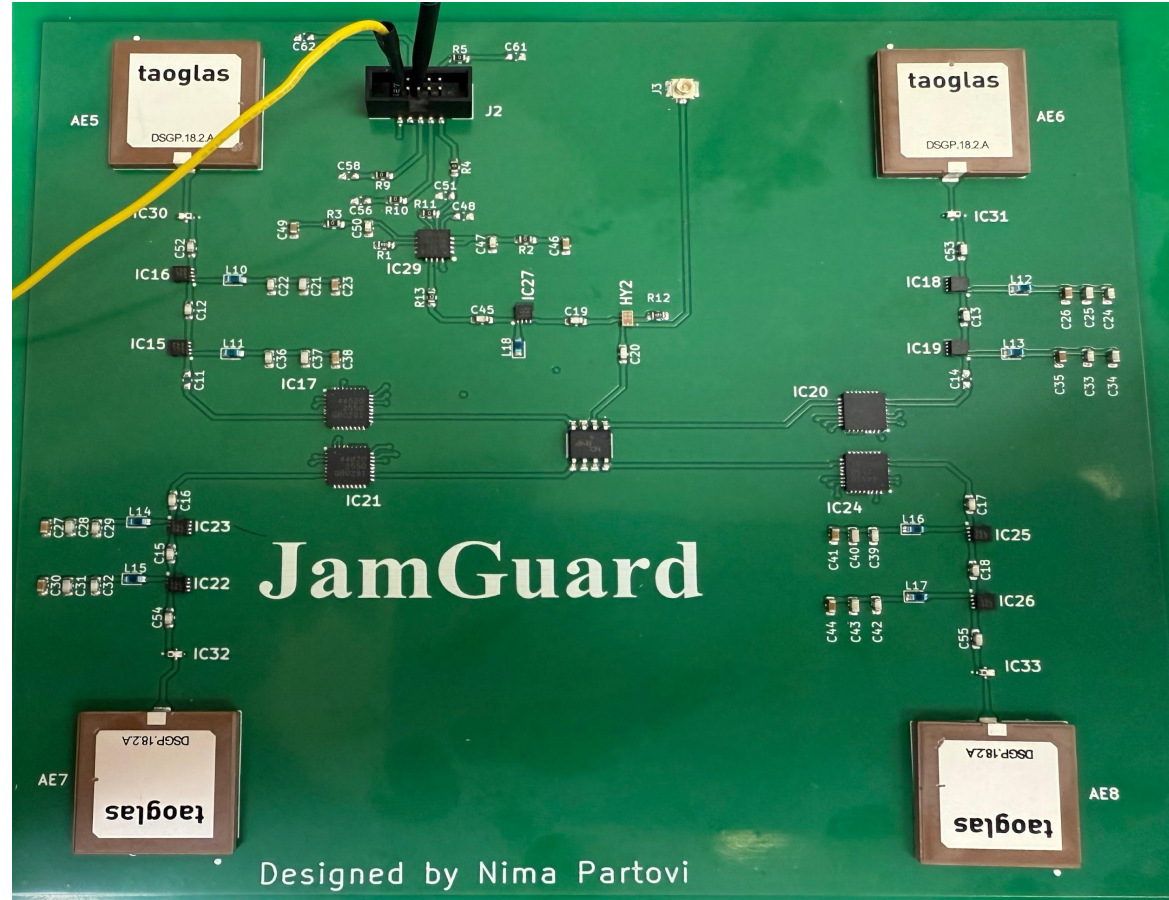


Phased Array

- A phased array uses multiple antennas working together
- Each element transmits the same signal with a slight phase shift
- These phase shifts control the radiation pattern
- **Constructive interference**
Signals **add up** in a desired direction (main beam)
- **Destructive interference**
Signals **cancel out** in other directions (nulls)
- **Result**
We can **steer the beam electronically** (no moving parts)



- Replace the current antenna on a GPS system with Jam Guard
- Integrate a null forming system into the system to block/reduce jammer signal before getting to the GPS receiver



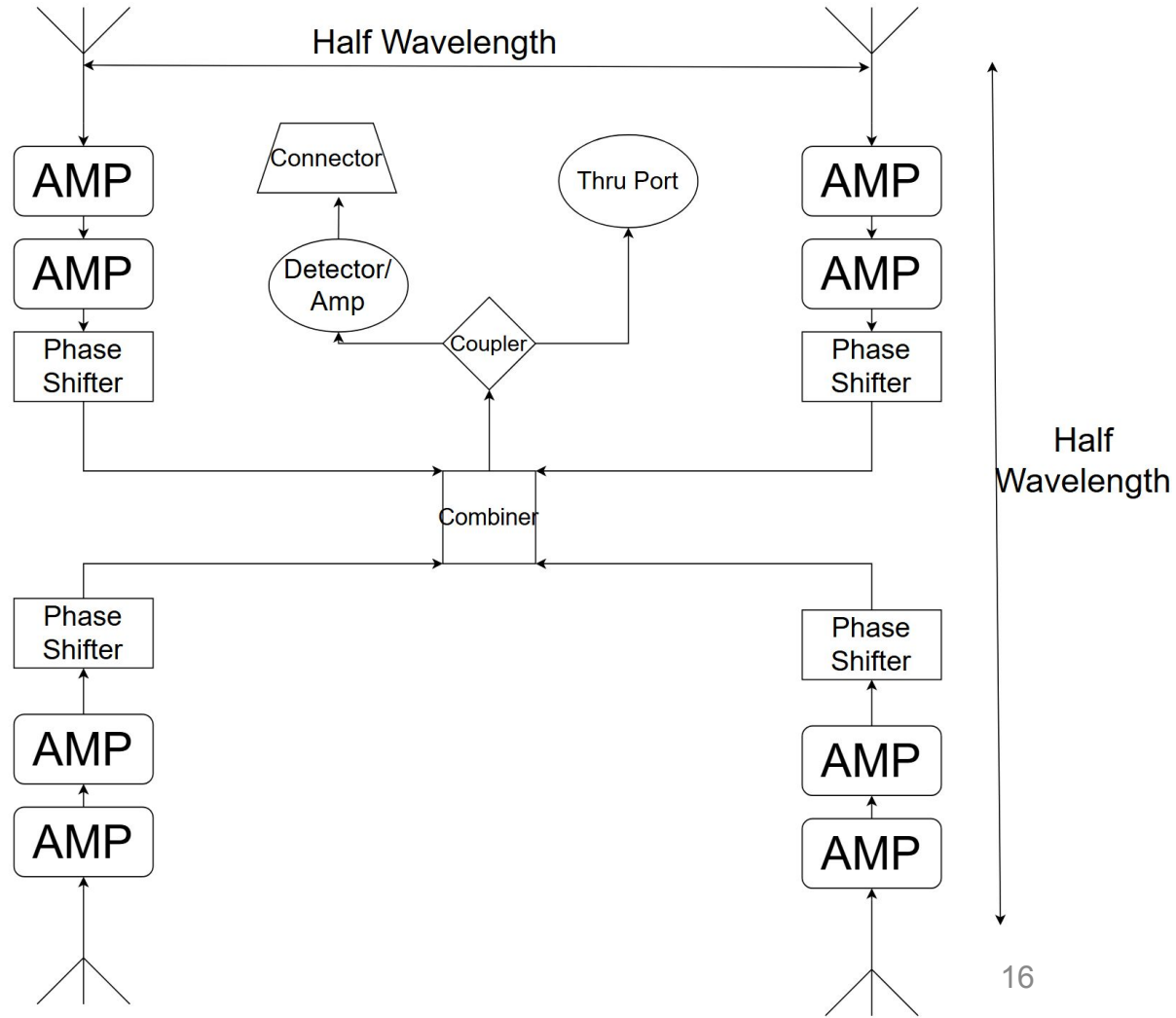
PCB layout (Block Diagram)

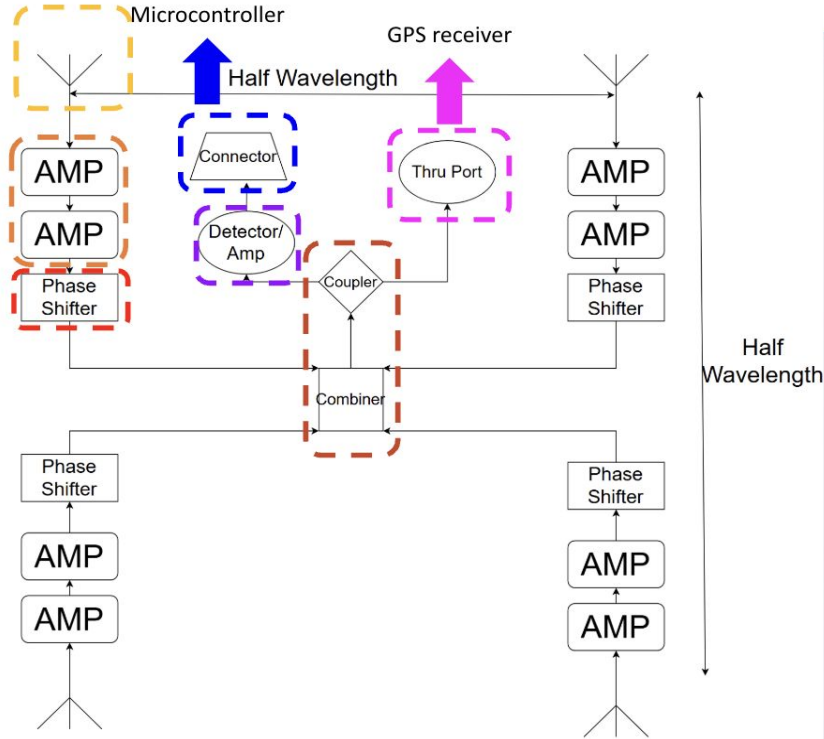
$$c = f\lambda$$

$$C = 3 * 10^8 \text{ m/s}$$

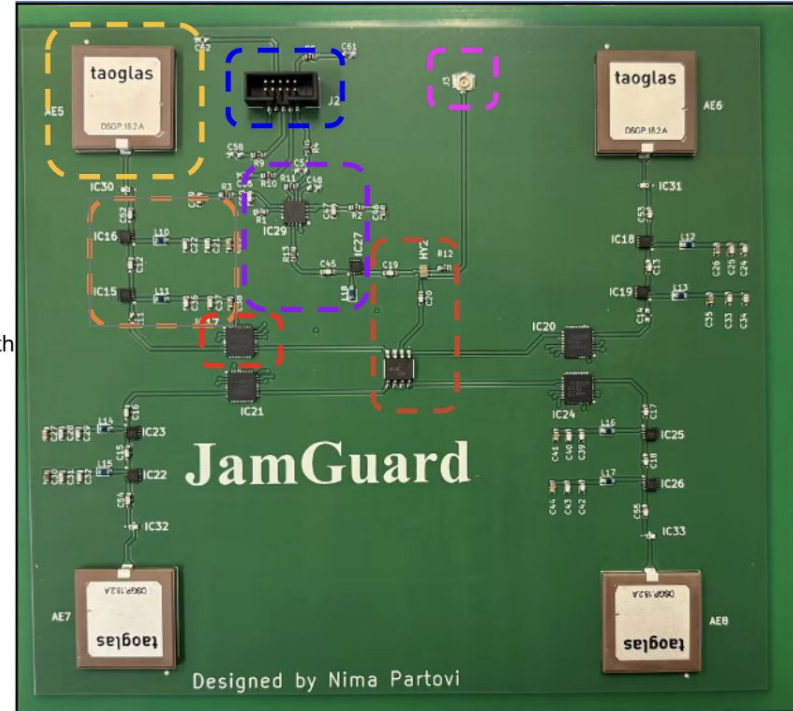
$$f = 15.7542 * 10^8 \text{ s}^{-1}$$

$$\lambda = 19.5 \text{ cm}$$



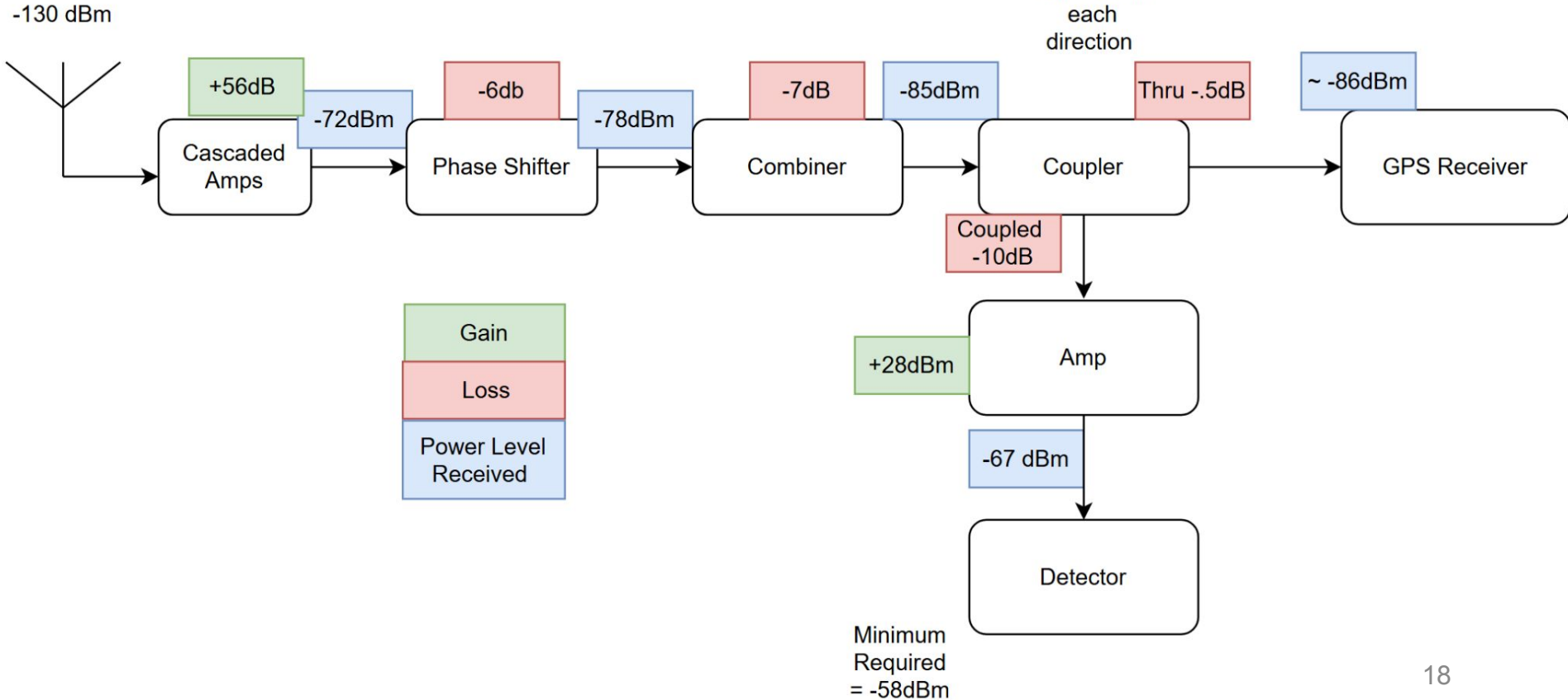


System block diagram



Fabricated Jam Guard PCB

Link Budget



$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \frac{F_5 - 1}{G_1 G_2 G_3 G_4} + \frac{F_6 - 1}{G_1 G_2 G_3 G_4 G_5}$$

1. Filter
2. Amp 1
3. Amp 2
4. Phase shifter
5. Combiner
6. Coupler

$$F(\text{total}) = \sim 1.6$$

$$NF = \sim 2.15 \text{ dB}$$

Beamforming & Null Steering Research

- Compute phase shifts for each antenna element
- Radiation pattern modeled using array factor:

$$AF(\theta, \phi) = \sum_{m=0}^1 \sum_{n=0}^1 e^{j[kd(m \sin \theta \cos \phi + n \sin \theta \sin \phi) + \phi_{m,n}]}$$

Null Depth & Null Width Experiments

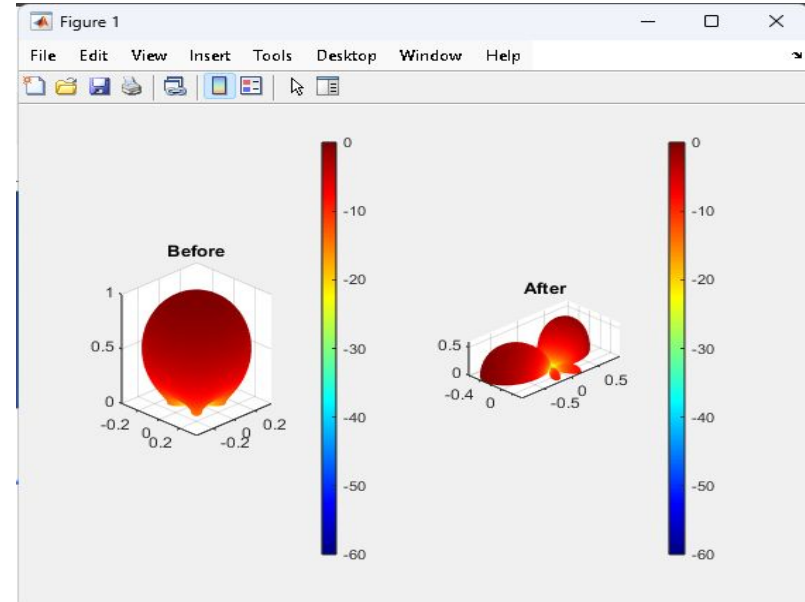
- Studied null depth vs phase resolution (5° steps)
- Identified limits of scan-based approaches

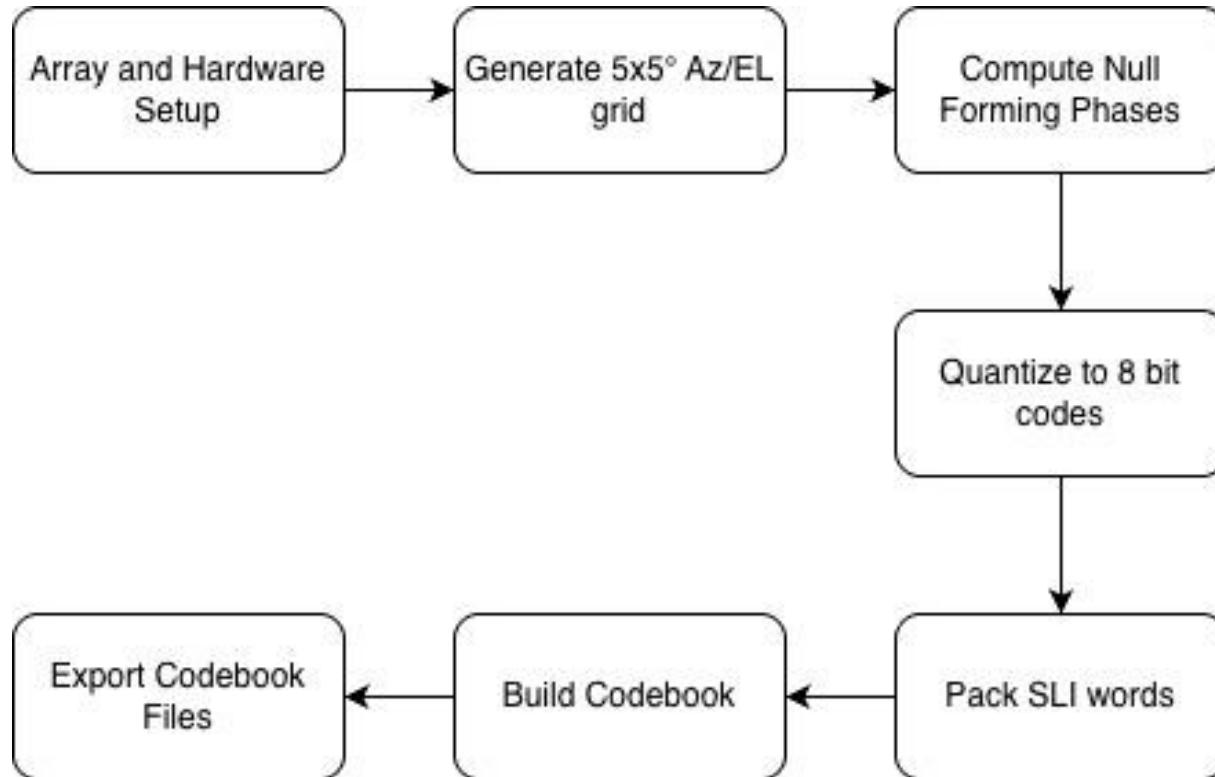
Identified Limitations of Scan-Based Approaches

- Transitioned to **codebook-based null steering**
- Precomputed phase values for fast real-time response

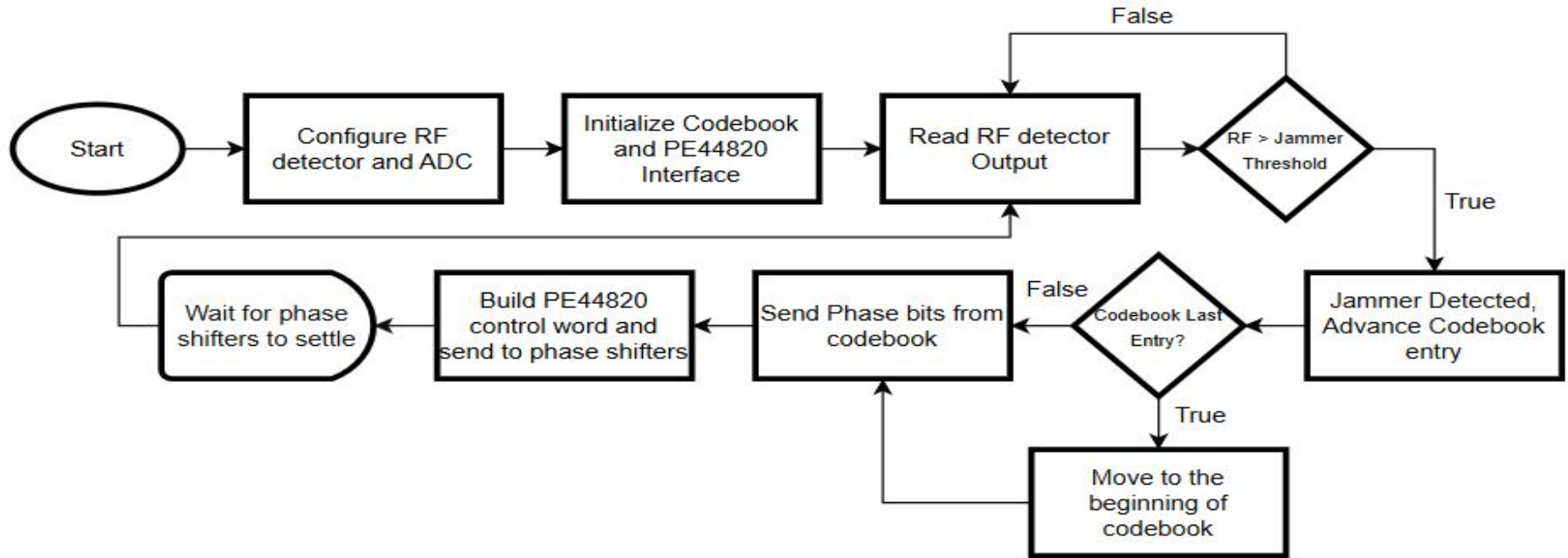
Steering away from Jammer

- Developed a **2x2 phased-array system** for directional null steering
- Generated a **3D radiation pattern** to visualize beam shape and null formation
- Built a **5° × 5° azimuth/elevation codebook** for predefined jammer directions
- Computed phase shifts to place a **null at jammer direction**
- Quantized phases into **8-bit PE44820 hardware control codes**
- Exported a **lookup-table codebook** for real-time SI based control





Software Flowchart



Design Matrix - Microcontroller

	Weight	PIC18F47K42	STM32 NUCLEO-L432KC	ESP8266 Node-MCU	ESP32 WROOM-32E	PYNQ -Z2
Cost	0.175	0.24	0.18	0.31	0.25	0.02
Size	0.075	0.26	0.30	0.12	0.28	0.14
Clock Speed	0.100	0.12	0.18	0.22	0.26	0.22
Software Compatibility	0.100	0.18	0.22	0.20	0.24	0.16
Power Consumption	0.200	0.32	0.28	0.08	0.14	0.08
Processing Speed	0.100	0.14	0.22	0.18	0.26	0.20
Availability/ Familiarity	0.300	0.36	0.22	0.18	0.22	0.10
Score		0.27	0.24	0.18	0.24	0.13

Design Matrix - Power Detector

	Weights	AD8318	ADL5513	ADL5902	LTC5507ES6
Cost	0.047	0.25	0.17	0.17	0.42
Freq. Coverage	0.193	0.28	0.22	0.28	0.22
Dynamic Range	0.279	0.31	0.31	0.25	0.12
Bandwidth	0.240	0.25	0.25	0.19	0.31
Accuracy	0.144	0.27	0.27	0.33	0.13
Power Consumption	0.097	0.21	0.21	0.15	0.36
Score		0.27	0.26	0.24	0.23

Main Challenges

1. Developing Search Algorithm
2. PCB development including Selecting hardware/hardware cost/delivery and PCB fabrication
3. Performing RF based tests

Key Risks and Mitigation Plans

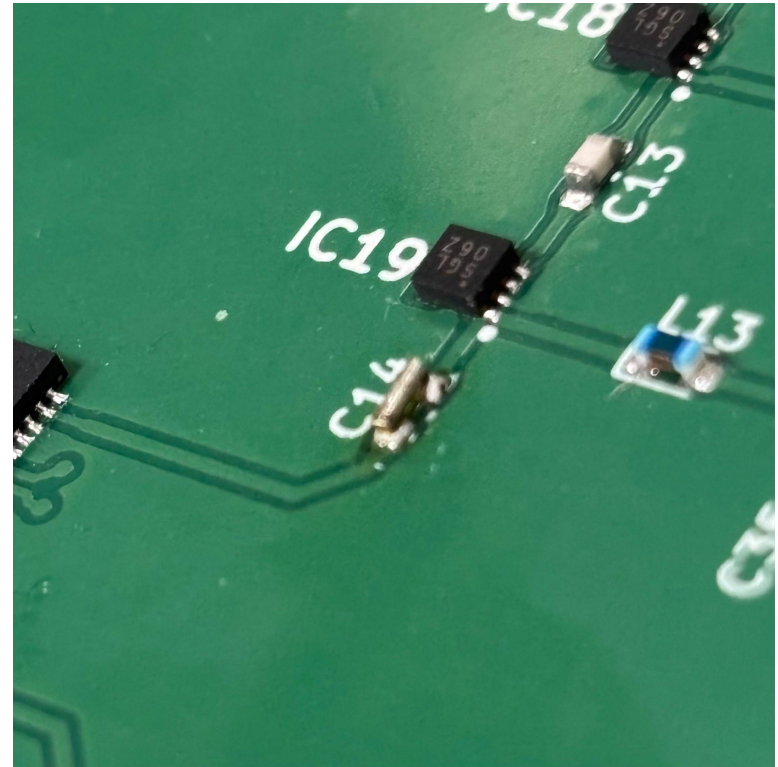
<u>Risk</u>	<u>Status</u>	<u>Mitigation Plan</u>
System requires multiple iterations beyond available time	Occurred, Not Resolved	Work with best iteration and proceed with testing
Specific RF hardware unavailable	Did not occur	Identified compatible components and validated them prior to integration
PCB wear and tear	Occurred, Resolved	Planned ahead and ordered multiple PCB boards and soldered missing components

PCB Fabrication - Missing Components

Problem: Upon inspection of the board, we noticed missing components near the phase shifter on 2 different antenna paths

Solution Plan: Soldered a tiny piece of resistor wire to complete the connection

Result: Successfully soldered on the resistor wire and made the connection, confirmed by continuity test



Summary of Tests Conducted

Test Number	Test Objective	Related ER(s)	Status	Notes
Pre-Test	Obtain GPS signal	N/A	Completed	Successful Lock-on
FT-1	Measure Jam Guard return loss	ER2.2	Completed	Obtained -13 dB return loss
FT-2	Jam Guard GPS Lock-on	ER2.1 ER6.1	Completed	Successful Lock-on
FT-3	Measure Size and Weight	ER4	Completed	Meets the defined specs

Summary of Tests Conducted

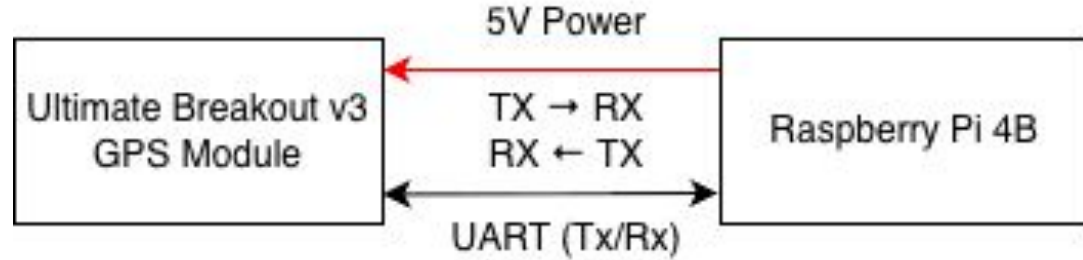
Test Number	Test Objective	Related ER	Status	Notes
FT-4	Measure Current Draw	ER5	Completed	Measured within spec (182 mA)
FT-5	Measuring InitializationTime	ER6.2	Completed	Measured within spec

Pre-Test GPS Receiver Board

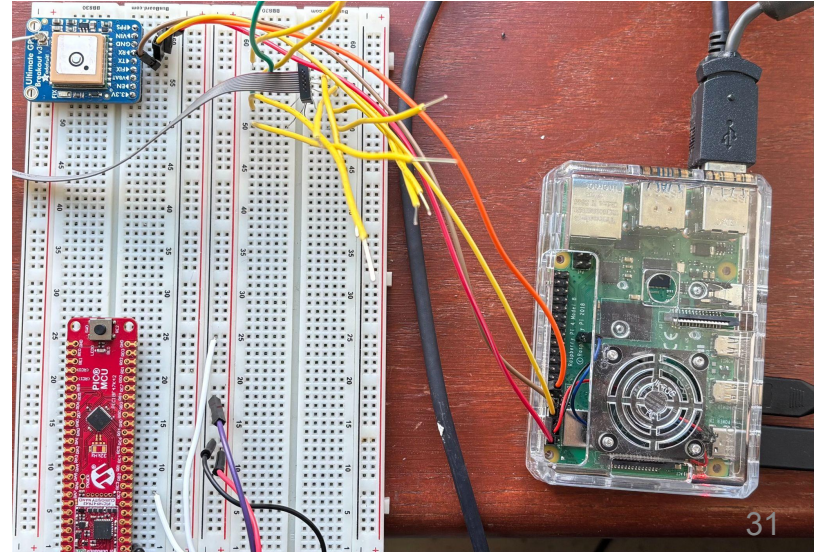
Objective: Confirm the Adafruit GPS receiver can maintain a consistent lock on

Procedure: Connect the GPS module to RPI via UART (Universal Asynchronous Receiver/Transmitter) and run python program to read data from the module

Conclusion: Ultimate GPS Breakout v3 GPS receiver functions properly with accurate longitude and latitude to our location after GPS lock on



```
STATUS: CONNECTED
TIME: 2026-04-24 09:27:19
LAT: 3820.4339 N
LON: 12240.4400 W
```

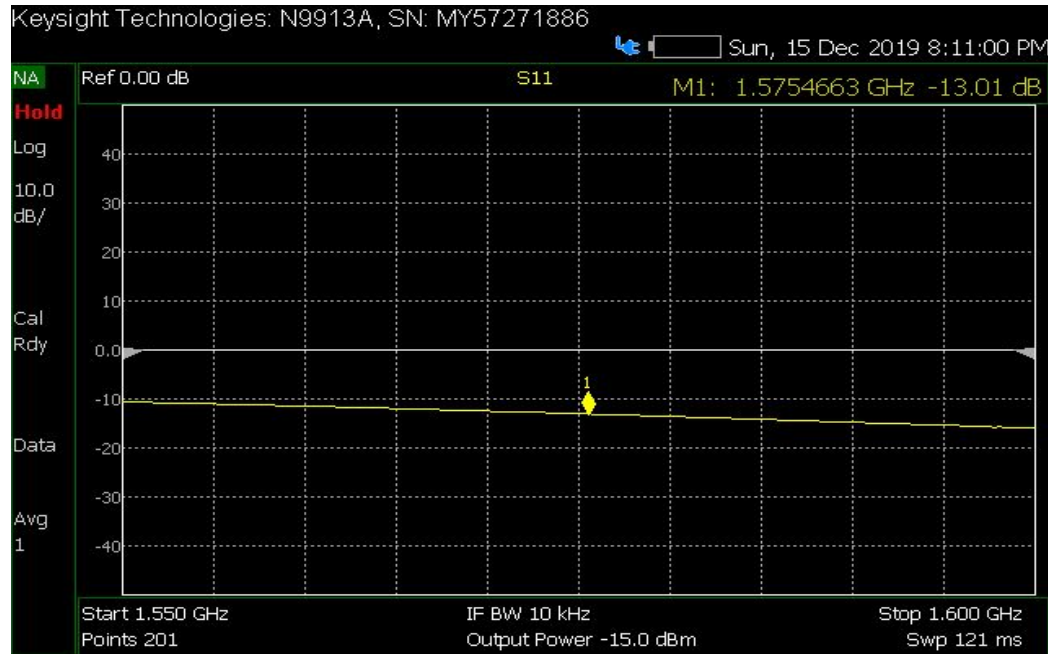
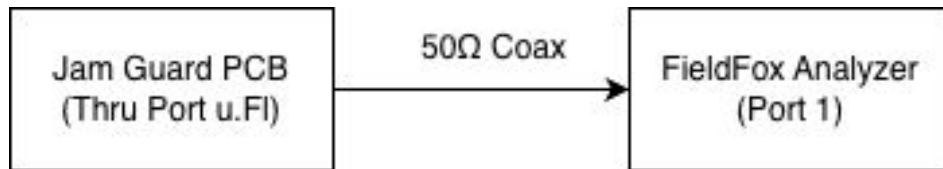


Objective: To confirm the RF return loss of Jam Guard is less than 10 dB (ER2.2)

Procedure: Use the RF Demo kit to calibrate the cable. Connect the calibrated cable to the thru port on Jam Guard. Measure S11 to see the return loss

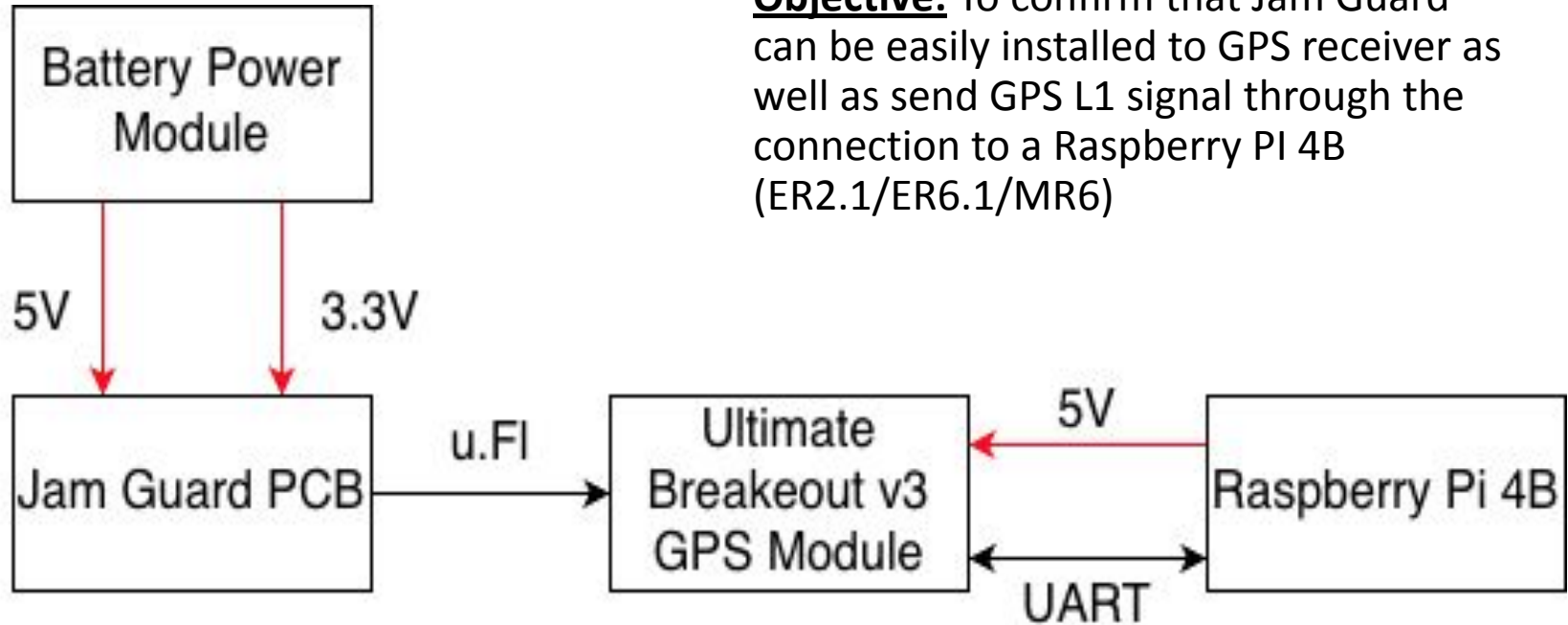
Conclusion: Jam Guard meets the requirement for less than 10 dB return loss

FT-1 Measuring Return Loss



FT-2 Jam Guard GPS Lock-on

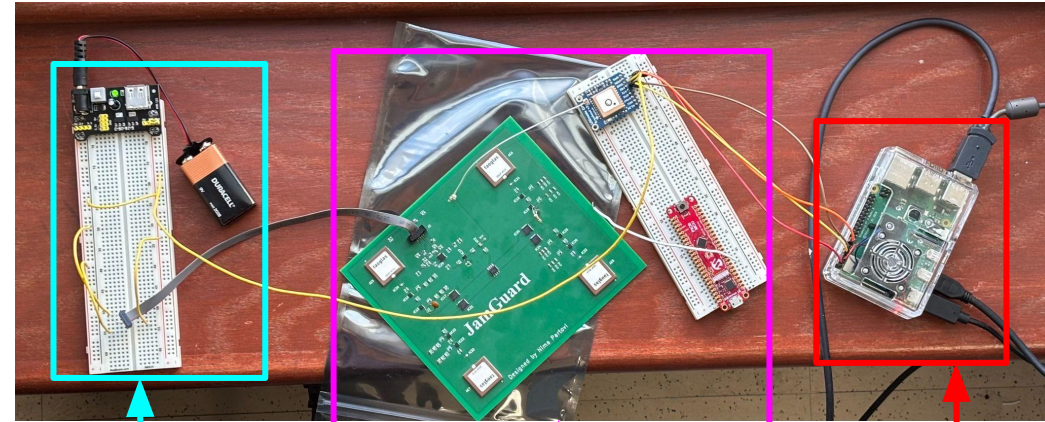
Objective: To confirm that Jam Guard can be easily installed to GPS receiver as well as send GPS L1 signal through the connection to a Raspberry Pi 4B (ER2.1/ER6.1/MR6)



FT-2 Jam Guard GPS Lock-on

Procedure: Connect the PCB to 5V and 3.3V power. Connect the board to the GPS module via [u.FI](#) cable. Power on the board and monitor response from the RPI

Conclusion: We confirmed that our PCB is able to receive GPS L1 and transmit it through the GPS module to the RPI



Power

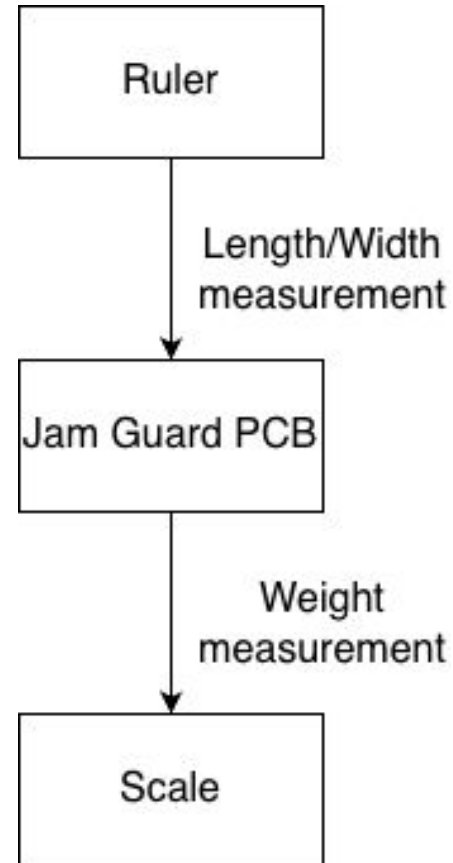
Jam Guard Connected to
GPS Receiver

RPI4B

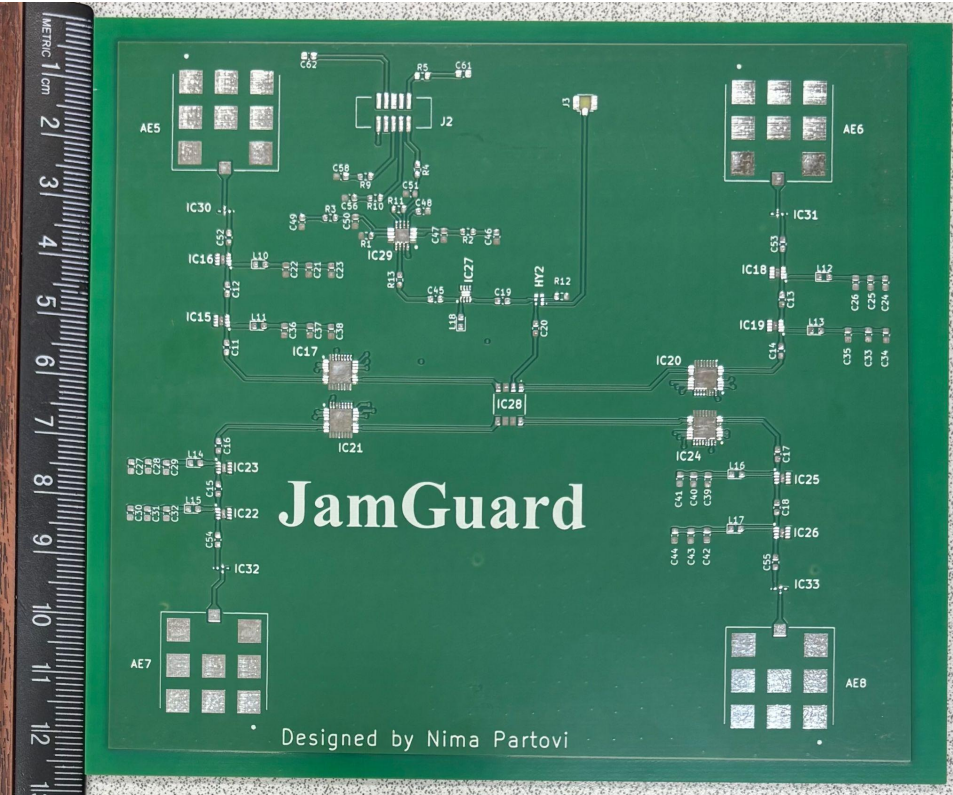
FT-3 Measuring Size and Weight

Objective: Determine if the Jam Guard PCB fits within 15x15 cm with weight of 600g (ER4/MR4)

Procedure: Measure the edges of the board and weigh it on a scale



FT-3 Measuring Size and Weight

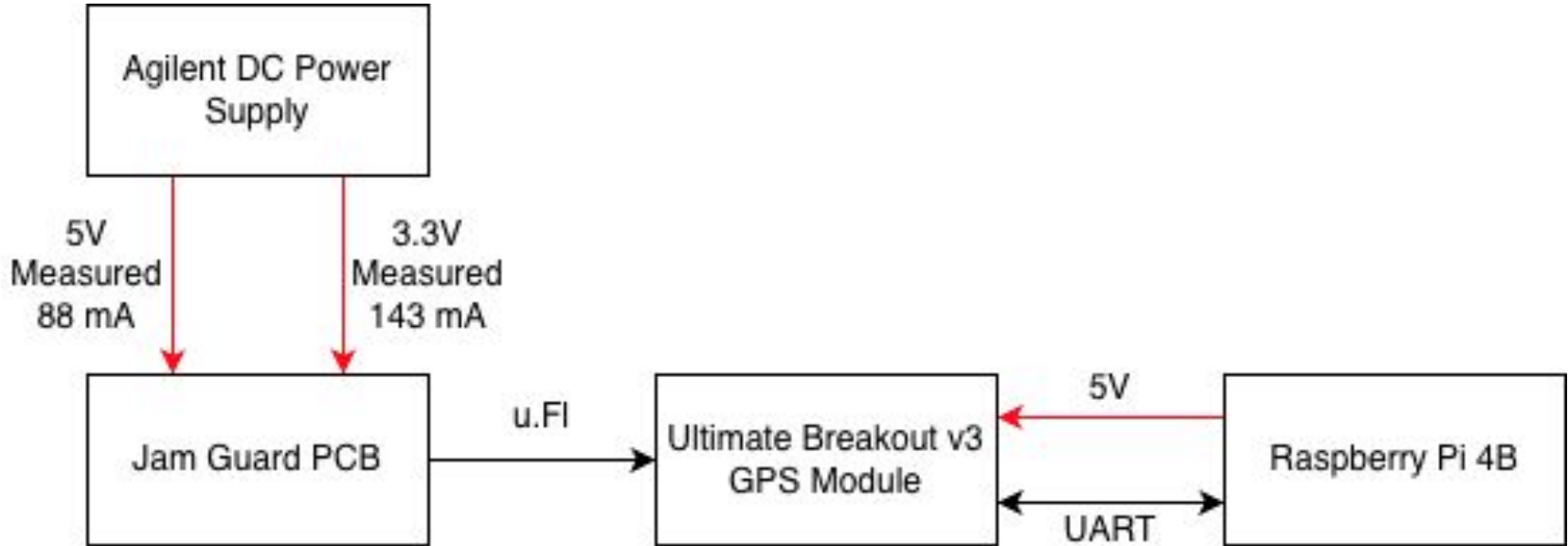


Length \approx 12.7 cm // Width \approx 14.4 cm



Weight = 80g

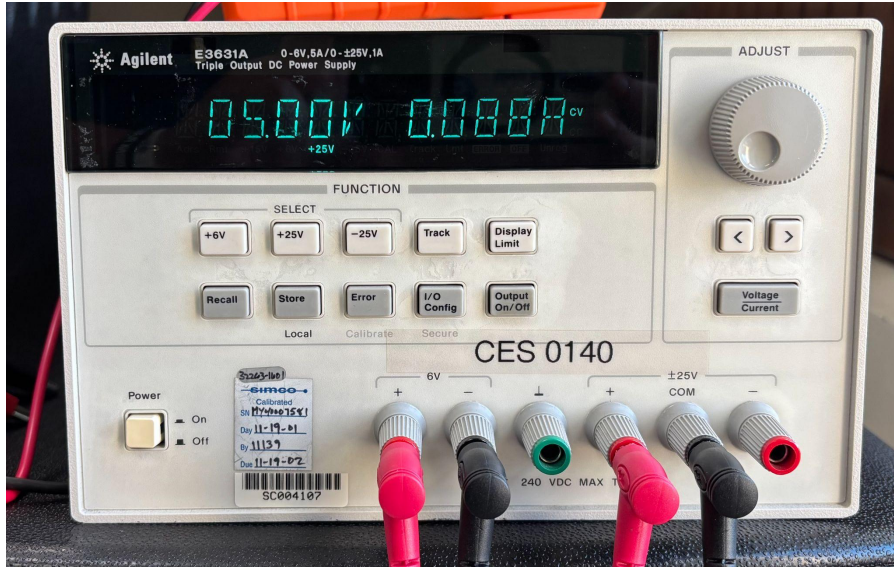
Conclusion: The Jam Guard PCB fits within the defined specs of length, width and weight.



Objective: Measure the total current being consumed by our device (ER5/MR5)

Procedure: Power Jam Guard using a bench supply (5V and 3.3V) and observe the current measurement shown on the supply

FT-4 Measuring Current Draw



$$P(5V) = 5 \times 0.088 = 0.44 \text{ W}$$

$$P(3.3V) = 3.3 \times 0.143 = 0.47\text{W}$$

$$P(\text{tot}) = 0.44 + 0.47 = 0.91\text{W}$$



$$I(\text{eq}) = (0.91/5) = 182 \text{ mA}$$

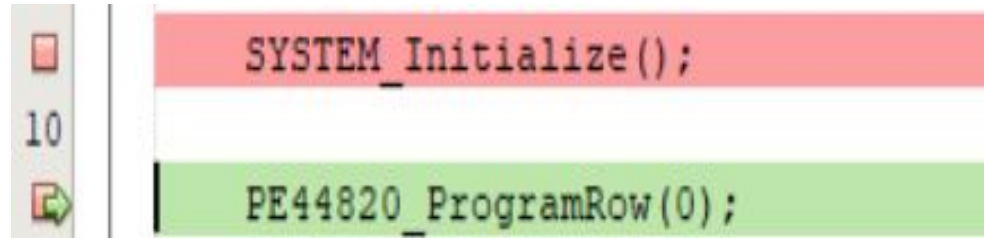
Conclusion: Jam Guard comfortably sits within the specs and met the requirement

FT-5 Measuring Initialization Time

Objective: To confirm the initialization time of the PIC is under $100\mu\text{S}$ (ER6.2)

Procedure: Run the code and measure the time between initialization and the first codebook row

Conclusion: Microcontroller is ready to scan under $41\mu\text{s}$ which meets the requirement



```
SYSTEM_Initialize();  
PE44820_ProgramRow(0);
```

Target halted. Stopwatch cycle count = 41 ($41\mu\text{s}$)

Component	Quantity	Price
Parts	2 Sets	\$319
(Phase Shifter)	4	(\$13 each)
(Antenna)	4	(\$11 each)
Assembly	2	Free(due to purchase size)
GPS Receiver		\$120(already owned)
Raspberry Pi	1	\$120
PCB	5	\$336
MISC		\$30-40
Taxes/Tariffs/Fees		\$246
Total		~\$1060

Initial Research/Concepts

PCB/Code Development

Testing/Optimization

Documentation

Ideal: September 2025
Actual: Mid November 2025

Ideal: Early February 2026
Actual: Mid March 2026

Ideal: Early April 2026
Actual: Ongoing

Ideal: Ongoing
Actual: Ongoing

Develop understanding of fundamentals and begin to start developing first blocks (conceptual layout/early code)

Get finalized forms of both PCB and Code for first implementation

Complete robust testing for use-cases and applications of JamGuard

Complete documentation of project

Key Issue: Dunning-Kruger

Key Issue: Lack of KiCAD experience

Key Issue: Late Dev of PCB and manufacturing/exporting issues

Past/Current:

EE 282 - Engineering Modeling Lab

EE310 - Introduction to Microcontrollers

EE 430 - Electromagnetics Theory and Applications

EE 444 - Introduction to RF Communications

EE 444L - Introduction to RF Communications

EE 470 - Introduction to Internet Of Things

Demonstration Video



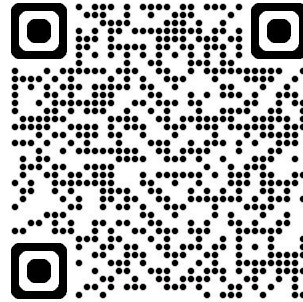
Board Improvements:

- Add onboard voltage regulators
- Idealize component spacing and fix missed board issues
- Correct Connector
- Achieve true 50Ω across RF chain

Future Testing:

- Implement “jammer” signal
- Increase compatibility testing
- Test null-forming code

Questions/Comments



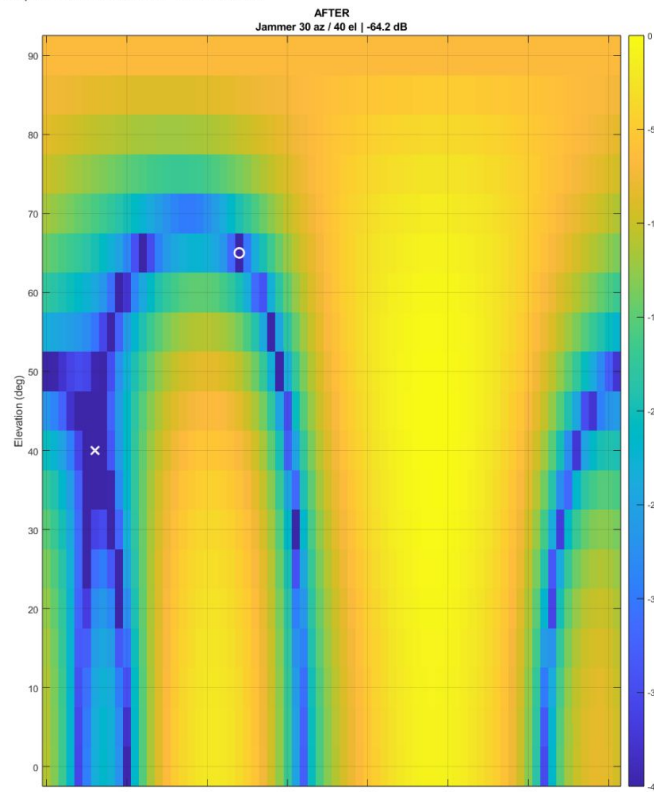
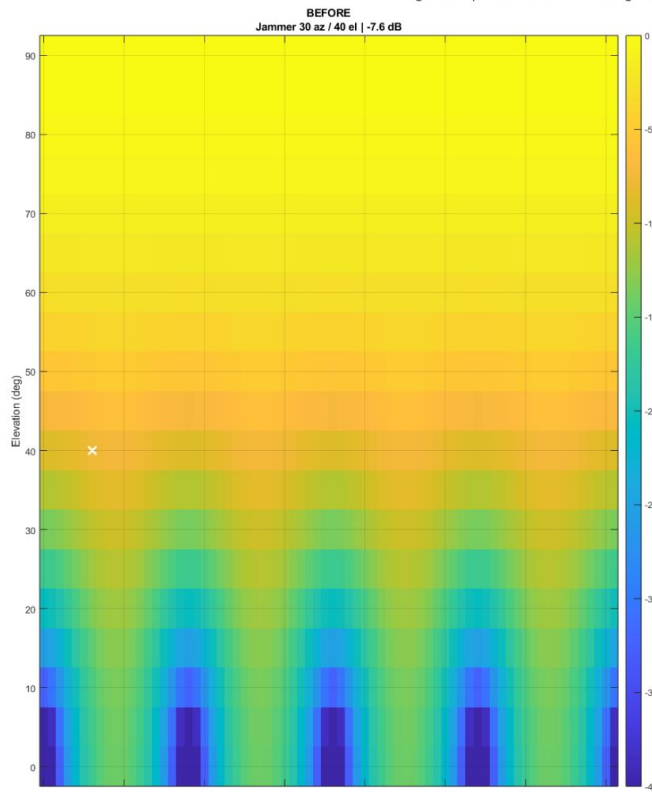
We want to acknowledge funding from the Center for Environmental Inquiry (CEI) at Sonoma State University: **The Norwick Memorial Fund**

Supporting Slides

Null Depth Example

GPU UTIL
GPU PWR
GPU TEMP
CPU UTIL
CPU TEMP

Nulling Result | Grid Resolution = 5 deg az x 5 deg el | Row = 123 | Deepest Null = -70.5 dB at 120 az / 65 el



Supporting Slides - Existing Problems

Google X

AI Mode **All** Shopping Images Videos Short videos News More Tools









Refine results **Vehicle Tracker** On sale Nearby Reviews For Sale Free Online Get


Price: Under \$90, \$0 - \$300, \$300 - \$700, Over \$700. Min Max Go

Sort by: Price: low to high, Price: high to low


Category: GPS Navigators, Security Cameras, Radio, Signal Finders, Color, Features

Popular products

 <p>GP5000 Car Anti-Tracking GPS Blocker, Navigation jammer \$119.99 Jammer Master Free delivery</p>	 <p>Portable Cell Phone Jammer Blocker \$659.99 Jammer Master</p>	 <p>CTL3520 Directional GPS Jammer Detector \$2,375.00 NavtechGPS</p>	 <p>Gps jammers for vehicles, Gps for vehicles, Red... \$17.63 Temu Free delivery on \$30+</p>
 <p>Suitcase Style Signal Jammer Blocker \$2,659.99 Jammer Master</p>	 <p>DIXSG Car Usb Gps Signal Interference \$28.69 neweggbusiness...</p>	 <p>65dB Anti-Jamming GPS \$358.00 OzRobotics.com Free delivery</p>	 <p>Desktop All-in-One Signal Jammer Block F... \$899.00 Jammer Master</p>



Jammer



GP5000 Car Anti-Tracking GPS Blocker, Navigation jammer
Images may be subject to copyright. Learn more

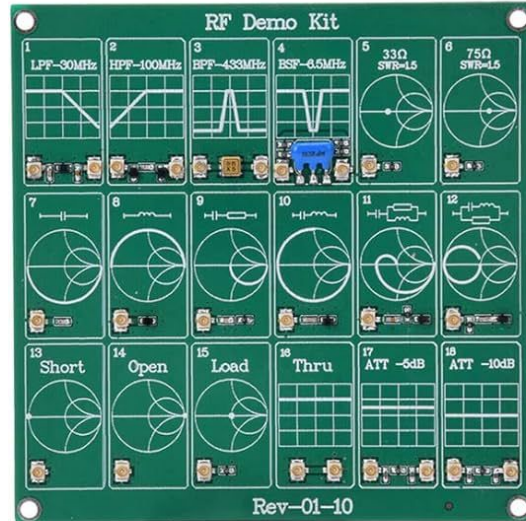
Typically \$120

Jammer \$119.99
GP5000 Car Anti-Tracking GPS Blocker, Navigation jammer
In stock online
Free delivery · 14-day returns

About this product

Brand: Jammer

Supporting Slides - RF Demo Kit





PCBWay Bom Quotation, Product No.:T-5D2W911214A

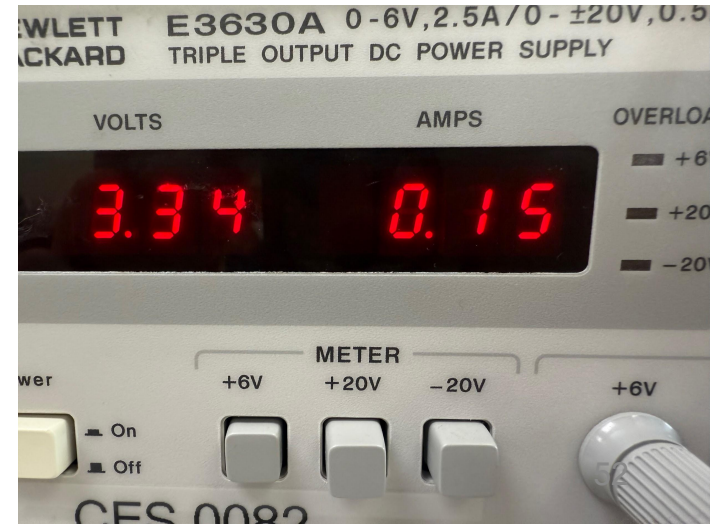
Item #	*Designator	*Qty	Description / Value	*Unit Price(2 units)	*Total	*Delivery Time	*Actual Purchase Mfg Part #	*PCBWay Note
1	AE5,AE6,AE7,	4	Antenna_Chip	\$11.677	\$93.416	7-10 Workdays		
2	C12,C13,C15,C	23	100 pF	\$0.011	\$0.506			
3	C21,C25,C28,C	8	1200 pF	\$0.071	\$1.136			
4	C23,C26,C27,C	8	1 uF	\$0.071	\$1.136			
5	C46,C49	2	.1 uF	\$0.060	\$0.240			
6	HY2	1	DC1722J5010AHF	\$3.303	\$6.606	7-10 Workdays		
7	IC15,IC16,IC18	9	SGL0622Z	\$0.839	\$15.102			
8	IC17,IC20,IC21	4	PE44820B-X	\$13.811	\$110.488			
9	IC28	1	BP4P1+	\$7.094	\$14.188			
10	IC29	1	AD8318	\$15.483	\$30.966		AD8318ACPZ-REEL7	Original part is not available, substitute we will supply is AD8318ACPZ-REE L7 (LFCSP-16(4x4)) ,pls confirm ok?
11	IC30,IC31,IC32	4	DPX162500DT-8014A1	\$0.968	\$7.744	7-10 Workdays		
12	J2	1	Conn_02x05_Odd_Even	\$1.223	\$2.446			
13	J3	1	Conn_Coaxial	\$0.408	\$0.816			
14	L10,L11,L12,L1	9	68 nH	\$0.123	\$2.214		LQW18AN68NG00D	Original part is not available, substitute we will supply is LQW18AN68NG00

Supporting Slides-Power Calculation

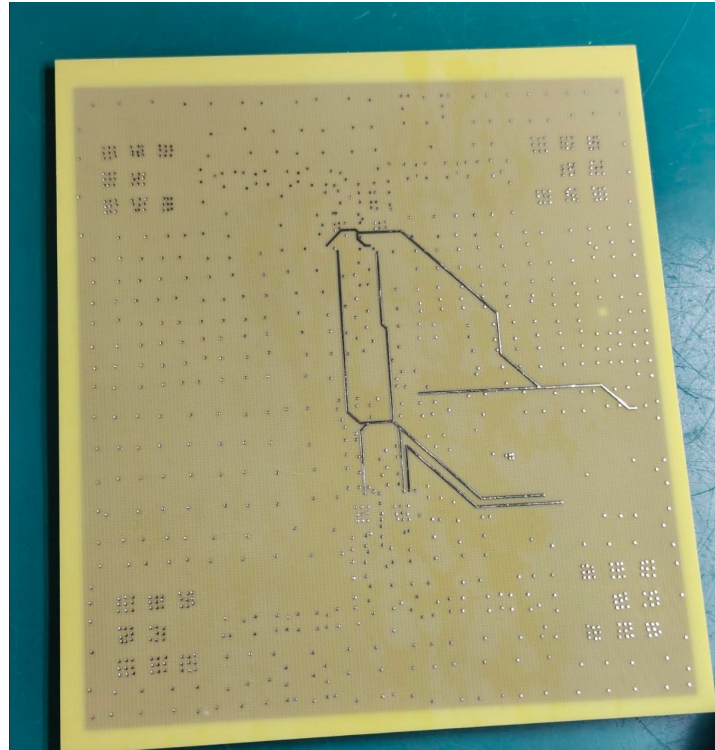
Expected Current 3.3V rail
9- SGL6022Z Amps - 9 mA
each

4-PE44820 Phase Shifters -
20 μ A each

Actual Current
3.3 V rail
- 150 mA



Supporting Slides-Manufacturing Troubles



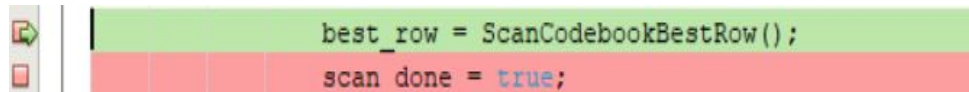
Here it is! Same idea, soldermask on back, if not specified should be full coverage

Extra FT- Worst case Scan time

Objective: Figure out the worst case scan time

Result: Worst case scan time is 3.822 ms

Conclusion: Some delays for RF settling increased the scan time



```
best_row = ScanCodebookBestRow();  
scan done = true;
```

|Target halted. Stopwatch cycle count = 3822 (3.822 ms)

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \frac{F_5 - 1}{G_1 G_2 G_3 G_4} + \frac{F_6 - 1}{G_1 G_2 G_3 G_4 G_5}$$

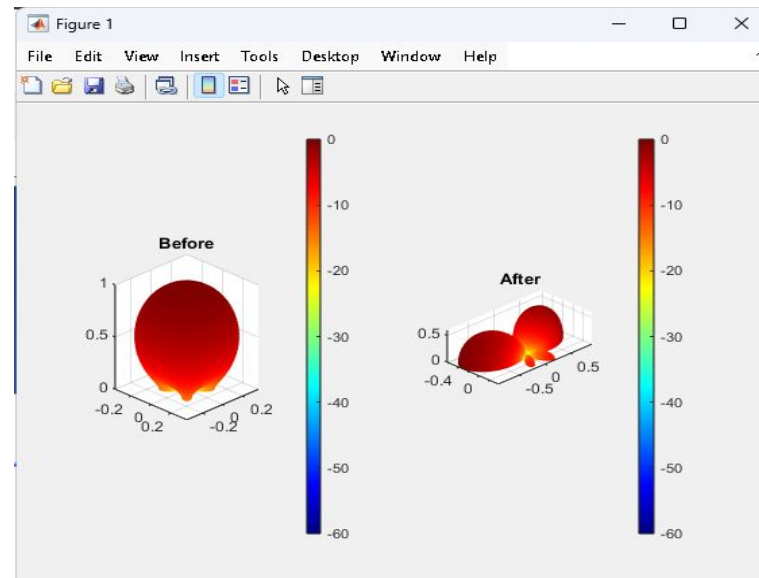
1. Filter
2. Amp 1
3. Amp 2
4. Phase shifter
5. Combiner
6. Coupler

$$F(\text{total}) = \sim 1.6$$

$$NF = \sim 2.15 \text{ dB}$$

- **Implemented full 2D array simulation with 3D pattern visualization**
- **Built $5^\circ \times 5^\circ$ lookup-table (codebook) for fast null steering**
 - Each entry stores phase settings for a specific jammer direction
- **Demonstrated before-and-after null patterns using codebook**

Achieved strong jammer suppression with phase-only control
- **Realistic constraints applied:**
 - Phase shifter step size
 - Array spacing (0.5λ)
 - Expected null width based on resolution



List of Components

1	Reference	Qty	Value	Manufacturer_Name	Manufacturer_Part_Number
2	AE5,AE6,AE7,AE8	4	Antenna_Chip	Taoglas Limited	DSGP.1575.18.2.A.02
3	C11,C12,C13,C14,C15,C16,C17,C18,C19,C20,C22	23	100 pF	Murata	GRM1885C1H101JA01D
4	C21,C25,C28,C31,C34,C37,C40,C43	8	1200 pF	Murata	GRM1885C1H122JA01D
5	C23,C26,C27,C30,C35,C38,C41,C44	8	1 uF	Murata	GRM188R71E105KA12D
6	C46,C49	2	.1 uF	Murata	GRM188R71C104KA01D
7	HY2	1	DC1722J5010AHF	ANAREN	DC1722J5010AHF
8	IC15,IC16,IC18,IC19,IC22,IC23,IC25,IC26,IC27	9	SGL0622Z	Qorvo	SGL0622Z
9	IC17,IC20,IC21,IC24	4	PE44820B-X	Peregrine Semiconductor	PE44820B-X
10	IC28	1	BP4P1+	Mini-Circuits	BP4P1+
11	IC29	1	AD8318	Analog Devices	AD8318ACPZ
12	IC30,IC31,IC32,IC33	4	DPX162500DT-8014A1	TDK	DPX162500DT-8014A1
13	J2	1	Conn_02x05_Odd_Eve	CNC Tech	3221-10-0300-00
14	J3	1	Conn_Coaxial	HiRose Electric	U.FL-R-SMT-1
15	L10,L11,L12,L13,L14,L15,L16,L17,L18	9	68 nH	Murata	LL1608-FSL68NJ
16	R1	1	499	Yageo	RC0603FR-07499RL
17	R2,R3,R4,R5,R6,R7,R9,R10,R11	9	0	Yageo	RC0603JR-070RL
18	R12	1	49.9	Yageo	RC0603FR-0749R9L
19	R13	1	52.3	Yageo	RC0603FR-0752R3L

Likelihood → Consequences ↓	Rare (1)	Unlikely (2)	Possible (3)	Likely (4)	Certain (5)
Catastrophic (5)	Jamming Test damages Hardware (5)	Complete PCB Failure (10)	Testing Environment Unavailable (15)	Specific RF Hardware Unavailable (20)	System Requires Multiple Iterations Beyond Available Time (25)
Major (4)	Algorithm Produces Unstable Outputs (4)	RF Interference Degrades Testing Results (8)	Algorithm Failure (12)	PCB Fabrication Issues (16)	Multiple Debug Phases Slow Integration (20)
Moderate (3)	Soldering Components Onto PCB (3)	Incompatibility between software and hardware (6)	Learning RF Measurement Tools (9)	Continued Algorithm Development (12)	Calibration Drift Over Time (15)
Minor (2)	Hardware Wear and Tear (2)	Tooling or Scripting Errors (4)	Substitute Components (6)	Setting Up Testing Plans (8)	More Testing Than Anticipated (10)
Negligible (1)	Formatting Issues (1)	Communication Issues in Group (2)	Project Organization (3)	Meetings With Advisors (4)	Time Spent Testing (5)

where :

- m, n : antenna index in the x - and y -directions
- k : wave number ($2\pi/\lambda$) where λ is the wavelength
- d : distance between two adjacent antennas
- θ : elevation angle
- ϕ : azimuth angle
- $\phi_{m,n}$: phase shifter phase for antenna at (m, n)

References

- IEEE Guide for